Optimal position control of synchronous reluctance motor via totally invariant variable structure control

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Abstract: A newly designed optimal control method is presented. The proposed controller is designed via combining classical state feedback control and variable structure control (VSC). This new method fully matches the merits of the easy design of the linear quadratic (LQ) method and the strong robustness of the VSC. The presented optimal control method is demonstrated on a synchronous reluctance motor (SynRM). It is proved that the synchronous reluctance motor can be used in position control by the proposed method, and the designed performance can be easily obtained regardless of the disturbance and uncertainty. A prototype PC-based SynRM control system is built to verify the validity of the proposed scheme.

1 Introduction

In the past decade, variable structure control (VSC) strategies have been the focus of many studies and research into control of the AC servodrive system [1, 2] because the VSC can offer many properties, such as insensitivity to parameters variations, external disturbance rejection and fast dynamic response.

Generally speaking, to design a conventional sliding mode control (SMC) system, there are two design phases that must be considered; the reaching phase and the sliding phase. The robustness of a VSC system resides in its sliding phase, but not in its reaching phase. In other words, the closed-loop system dynamics are not completely robust all the time. In addition, while the designed techniques for the sliding mode are well established, it is not easy to shape the dynamics of the reaching phase. Moreover, the design of the sliding mode is in the reduced-order system, which is not available and straightforward for the designer to implement.

From the designer's viewpoint, linear state feedback control is theoretically an attractive method for controlling a linear plant represented by a state-space model. The method has the full flexibility of shaping the dynamics of the closed-loop system to meet the desired specification. Techniques such as pole placement or the linear quadratic method can be used to achieve the designed aims. Usually, the motor system can be modelled as a second-order state-space system where the mechanical velocity and position are used as the system states. This method seems well suited to the motor system. However, there are few real motor systems adopting this method as the controller design. The main problem is, while the desired system response can be achieved in the nominal system, it is difficult to incorporate robustness considerations into the design procedure.

However, to consider the optimal performance requirement, the linear quadratic (LQ) method is an easy way to design the control law. The LQ method is based on the state-space model. To find the control law, the Riccati equation must be solved, then an optimal feedback gain will be obtained naturally. Regardless of the regulator problem or the tracking control problem, control under this feedback gain will lead to a minimum performance index. Despite the facts, once the external disturbance and/or the parameter uncertainty occur, then the performance may not be obtained just like the state feedback control. First, the system with parameter uncertainty will result in a response without matching the predesigned state trajectories, and the integral of the optimum performance index cannot be obtained. Secondly, when the system is subjected to an external disturbance, the system states will not tend to zero. Then, it may be impossible to achieve a steady-state error with a control u, which is from the state feedback control, tending to zero. Therefore, the integral of the performance index will become infinite when time approaches infinity. Like the conventional proportional-plus-integral (PI) control, one strategy to force the system states to zero is to use integral feedback [3]. On the contrary, the resultant system is an augmented system the order of which is increased by one, and the poles are located at different positions without matching the desired position. Therefore, the responses will be different from the originally designed optimal requirements.

The synchronous reluctance motor (SynRM) has long been regarded as inferior to other types of AC machine and has been used only for variable-frequency applications with open-loop control, such as in fibre spinning machines and pumps. However, compared to other types of AC machines, synchronous and induction motors, the synchronous reluctance motor has advantages in many applications because of the simplicity of its construction and control. For example, no slip rings, brushes, DC field windings are required as for a synchronous motor. No computation of the slip is needed for high-performance servodrive as needed for an induction motor, and it has high efficiency and low cost when compared with most servomotors.

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Recently, because of the advantages mentioned above and tremendous progress in machine design and power electronics, many researchers have devoted time to studying the control of synchronous reluctance motors [4–6]. In addition, to consider the torque control for SynRM, four control methods were introduced by Betz *et al.* [7]. Different torque control methods have the attributes belonging to their definitions and fields of application. To consider power dissipation, Matsuo *et al.* [8] presented the current vector control method to improve the control efficiency. For speed control, Liu and Lin [9] presented a speed control scheme by combining sliding mode control and fuzzy control. Even so, there are still a few researchers focusing their attention on the position control of SynRM using modern control strategies.

In this paper, we have developed an effective optimal control strategy for the position control of the synchronous reluctance motor using a newly designed method. The proposed position control scheme, based on the totally invariant variable structure control [10], can fully match the above mentioned requirements and solve the problems of SynRM position control. In the control scheme, maximum torque control for SynRM is adopted to generate the required torque, and the developed control scheme possesses the full flexibility of state feedback control in shaping the closed-loop dynamics using conventional state feedback, and the feedback gain is designed, in the same way as the designed procedure of the linear quadratic method, by solving the Riccati equation. Furthermore, on the basis of the newly designed controller, the position control system will keep in the sliding phase at the beginning and throughout the control process. Thus, the system is robust and invariant for all the control process. Because the proposed controller is invariant, the designed position of poles can be conserved to achieve the optimal performance requirement whether the perturbations and uncertainties exist or not.

2 Linear optimal control method and totally invariant variable structure control method

In this Section, based on the state-space equation of the linear system and the introduced performance index, the general control concepts of the linear quadratic method will be described. Thereafter, a modified linear quadratic method, which is in a sense integral feedback control, will be introduced to reduce the effect of nonzero steady-state error caused by the external disturbance. At the same time, a newly defined performance index is co-ordinated with this modified quadratic method. In the following Section, the proposed totally invariant variable structure control [10] will be described. Then, according to the theorem of the linear quadratic method and the proposed new method, one will have a controller which not only conserves the property of the linear quadratic method but also is independent of parameter uncertainty and external disturbances.

2.1 Linear quadratic method

When designing a linear control system to satisfy the dynamic system requirement, pole placement is an adequate way to meet this objective. If an optimal performance index is also considered, the linear quadratic method is easily able to determine the desired feedback gain to satisfy the requirement.

In view of the linear quadratic optimal control for a single-input system

$$\dot{x} = Ax + bu \tag{1}$$

where A is a $n \times n$ matrix, b is a $n \times 1$ vector, x is the $n \times 1$ state vector and u is the scalar control, respectively. A performance index J_1 is first defined to be of the form

$$J_1 = \int_0^\infty (x^T Q x + r u^2) dt \tag{2}$$

where r is a positive constant and Q is nonnegative definite. Via the linear quadratic method, to yield the optimal control law in an infinite period, the Riccati equation

$$A^T P + PA - r^{-1} P b b^T P + Q = 0 \tag{3}$$

must first be solved. Let $\bar{P}$ be the solution for eqn. 3 and be nonnegative symmetric. Thus, the control law to yield a minimum performance index is as follows:

$$u = -(r^{-1} b^T \bar{P}) x \triangleq -k^T x \tag{4}$$

where k is the feedback gain.

2.2 Modified linear quadratic method

To consider a control system which is designed according to the linear quadratic method, the steady-state error will occur if an external disturbance exists. To reduce the steady-state error caused by the external disturbance, the integral feedback [3] is an appropriate alternative. The integral feedback system has a similar structure to Fig. 1. This control system is an augmented system, where an integrator is inserted between the original system and the linear constant controller. To consider this augmented system, the performance index J_1 is redefined as

$$J_2 = \int_0^\infty (x^T Q x + r u^2 + s \dot{u}^2) dt \tag{5}$$

where s is a positive constant. Similar to the preceding design procedure for the minimum requirement of J_1, one should find a control law to minimise the performance index (eqn. 5) under the constraint of the augmented system (Fig. 1.)

To solve this problem, one defines the new state vector and control as

$$x_1 = \begin{bmatrix} x \\ u \end{bmatrix}, u_1 = \dot{u} \tag{6}$$

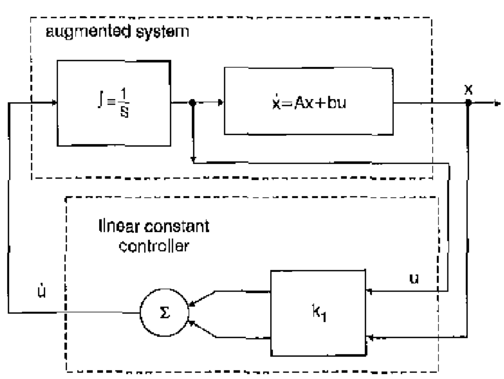

Fig. 1 *Modified LQ method with integral feedback*

and the new matrices

$$F = \begin{bmatrix} A & b \\ 0 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} Q & 0 \\ 0 & r \end{bmatrix} \tag{7}$$

Now, in terms of the newly defined variables, the modified system (Fig. 1) and performance index (eqn. 5) can be written as follows:

$$\dot{x}_1 = Fx_1 + gu_1 \tag{8}$$

$$J_2 = \int_0^\infty (x_1^T Q_1 x_1 + su_1^2)dt \tag{9}$$

Then similarly to the preceding method to obtain the optimal control law, the Riccati equation

$$F^T P + PF - s^{-1} Pgg^T P + Q_1 = 0 \tag{10}$$

is solved and the solution $\bar{P}$ of (eqn. 10) is used to determine the control law u_1

$$u_1 = -(s^{-1} g^T \bar{P})x_1 \stackrel{\Delta}{=} -k_1^T x_1 \tag{11}$$

2.3 Totally invariant variable structure control method

It is obvious that the modified linear quadratic method has the system order increased by one due to the inserted integrator. This strategy is able to reduce the steady-state error to zero. However, it also slows down the response compared to the design of the linear quadratic method. Furthermore, for both the linear quadratic method and the modified linear quadratic method, once parameter uncertainty is present, the responses will not be conserved as the nominal condition.

To conserve the responses and reject the effects of the external disturbance and parameter uncertainty, the proposed totally invariant variable structure controller will guarantee them. The control concepts of the totally invariant variable structure controller are as follows.

Considering also the single-input linear system (eqn. 1) in its nominal condition and expressed in the controllable canonical form

$$\dot{x} = Ax + bu \tag{12}$$

where

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -a_1 & -a_2 & \dots & -a_n \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}$$

In (eqn. 12), constant b in $\boldsymbol{b}$ is assumed to be positive.

Under the perturbed condition (eqn. 12) still in the canonical form, it becomes

$$\dot{x} = (A + \Delta A)x + (b + \Delta b)u + d \tag{13}$$

where ΔA and Δb are the perturbations in A and b, respectively, and $d \in R^n$ represents the external disturbance. To maintain the controllability, it is assumed that $(b + \Delta b) > 0$. (Eqn. 13) can be expressed in the form

$$\dot{x} = Ax + bu + p \tag{14}$$

$p \in R^n$ is the total perturbation given by

$$p = \Delta Ax + \Delta bu + d \tag{15}$$

Let the system be under linear state feedback control u_L, that is

$$u_L = -k^T x, \qquad k^T = [k_1 \quad k_2 \quad \dots \quad k_n] \tag{16}$$

where k is feedback gain that can be obtained using a preferred linear control design technique, such as pole placement or the linear quadratic method. The closed-loop dynamic, in the nominal condition, is given by

$$\dot{x} = [A - bk^T]x = A_c x \tag{17}$$

with

$$A_c = A - bk^T = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -\alpha_1 & -\alpha_2 & \dots & -\alpha_n \end{bmatrix} \tag{18}$$

where

$$\alpha_i = a_i + bk_i, \qquad i = 1 \text{ to } n \tag{19}$$

Next, consider a scalar function

$$\sigma(x, t) = c^T[x - x_0] - c^T A_c \int_0^t x(\tau)d\tau \tag{20}$$

Based on (eqn. 17), $\sigma(x, t) = 0$ under the nominal condition. Therefore, for any chosen state feedback (eqn. 16), the system possesses a sliding surface $\sigma(x, t) = 0$ on which the state slides.

It can be easily proved that the perturbed system (eqn. 14), under the condition $\sigma(x, t) = 0$, reserves an equivalent system dynamic as well as the closed-loop dynamic in the nominal condition given by (eqn. 17).

When perturbation p exists, the linear control u_L will not be able to maintain the sliding mode. Additional control effort is necessary to keep the states on the sliding surface. To control the states on the sliding surface under the perturbed condition, this extra control effort is given as $-q$ sgn (σ), then the resultant control is

$$u = u_L - q\ \text{sgn}(\sigma) = -k^T x - q\ \text{sgn}\ (\sigma) \tag{21}$$

The added term, $-q\,\text{sgn}(\sigma)$, is the variable structure control for the system and σ is its switching function. It is easy to prove that the choice of q in eqn. 21 can be given as

$$q = \frac{1}{b - \beta}\left[\sum_{i=1}^n \varepsilon_i |x_i| + d_m\right] \tag{22}$$

where $|\Delta b| < \beta$, $|\Delta a_i + \Delta bk_i| < \varepsilon_i$ and $|d| < d_m$. Thus, the curbing condition $\sigma\dot{\sigma} < 0$ is assured. Therefore, the closed-loop system dynamics for the nominal condition can be obtained i.e. the system will have its activity like $\dot{x} = A_c x$ regardless of the existence of disturbance and uncertainty. In view of eqn. 20, it is evident that $\sigma = 0$ at $t = 0$ and later. Thus, a system controlled by the proposed controller is in the sliding mode in the beginning, i.e. the system can have robust properties from the beginning of the control process.

Remark 1: The choice of q can be set as eqn. 22. However, if there is no prior estimation of uncertain parameters of $\Delta\alpha_i$, Δb and d, an adaptive law [11] can be used to estimate $|p|_{\max}$.

Remark 2: Note that totally invariant variable structure control, which is invariant to external disturbance and parameter uncertainty, is different from the conventional VSC system. For the conventional VSC, there are two phases, which indicate the hitting phase and sliding phase, existing in the control process and only the sliding phase can be controlled. Above all, the system controlled by conventional VSC is a reduced-order system. If the motor system is controlled by the conventional VSC-based controller, it will not show responses similar to those designed by pole placement. Furthermore, the robustness of the uncontrolled hitting phase cannot be guaranteed. But, the system controlled by totally invariant VSC is in the sliding mode in the beginning, the robustness can be guaranteed throughout the control process, and thus it is totally invariant. Specifically, the system's activity is still a second-order mode and can be designed by pole placement or by the linear quadratic method.

3 SynRM modelling

The d–q axes equations for SynRM are generally described as [7]

$$v_{ds} = L_{ds}\frac{d}{dt}i_{ds} + R_s i_{ds} - \omega_e L_{qs} i_{qs} \quad (23)$$

$$v_{qs} = L_{qs}\frac{d}{dt}i_{qs} + R_s i_{qs} + \omega_e L_{ds} i_{ds} \quad (24)$$

where v_{ds} and v_{qs} are the d-, q-axis stator voltages, i_{ds} and i_{qs} are the d-, q-axis stator currents, L_{ds} and L_{qs} are the d-, q-axis inductances, R is the stator resistance and ω_e is the electric frequency. The corresponding electromagnetic torque production is

$$\tau_e = \frac{3}{2}\frac{p}{2}(L_{ds} - L_{qs})i_{ds}i_{qs} \quad (25)$$

or

$$\tau_e = \frac{3}{4}\frac{p}{2}(L_{ds} - L_{qs})i_s^2 \sin(2\delta) \quad (26)$$

where p is the pole number of the motor; δ is the current angle; $i_s = \sqrt{i_{ds}^2 + i_{qs}^2}$ and

$$\begin{cases} i_{ds} = i_s\cos(\delta) \\ i_{qs} = i_s\sin(\delta) \end{cases}$$

The associated electromechanical equations are as follows

$$J_m\frac{d\omega_m}{dt} + B_m\omega_m = \tau_e - \tau_l \quad (27)$$

$$\frac{d\theta_m}{dt} = \omega_m \quad (28)$$

where θ_m is the rotor angular displacement, ω_m is the rotor velocity, J_m is the inertia moment and B_m is the damping coefficient.

There are four torque control strategies for SynRM. Three of them are constant power angle controls; maximum torque control (MTC), maximum power factor control (MPFC) and maximum rate of change of torque control (MRCTC). The last control strategy, constant current in inductive axis control (CCIAC), is a constant direct current control. In the following, a brief review of the MTC method is introduced.

For maximum torque control, the current angle is set at $\delta = 45°$. Since $\sin(2\delta) = \sin(90°) = 1$, eqn. 26 becomes

$$\tau_e = \frac{3}{4}\frac{p}{2}(L_{ds} - L_{qs})i_s^2 \quad (29)$$

or

$$\tau_e = K_T i_s^2 \quad (30)$$

where $K_T = (3/4)(p/2)(L_{ds} - L_{qs})$. The produced torque of MTC, eqn. 30, is always positive. To match the control methodology of variable structure control. two opposite

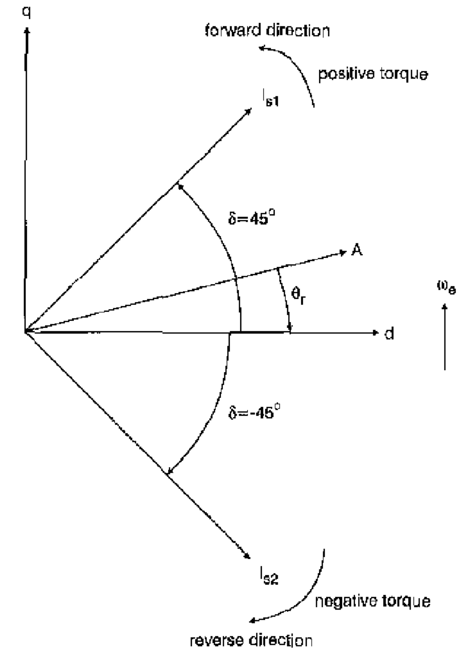

Fig. 2 *Positive and negative torque current vector of maximum torque control of SynRM*

control torques are necessary. One is positive to increase the motor shaft velocity, whereas the other is negative to decrease it. Therefore, eqn. 30 must be redefined to permit the production of negative torque. If the controller output is positive, it implies that the q-axis current vector must be placed ahead the d-axis 45° in accordance with the rotating direction, and we take it as a positive angle. When a negative torque is required, the power angle is placed behind the d-axis 45°, i.e. the current angle is set to $\delta = -45°$. In this mode, eqn. 30 can be modified as

$$\tau_e = -K_T i_s^2 \quad (31)$$

The corresponding concepts are shown in Fig. 2. It can be understood that if the MTC torque control strategy is used, one merely has to control the angle and magnitude of the current vector to match the desired torque.

As the concepts of torque control for SynRM have shown, MTC has the property of maximum torque per amp generation [7] and the convenience of deriving the desired torque. In addition, with consideration of both the power dissipation and system responses, the MTC method is adopted as the optimal performance control for SynRM torque control.

4 Optimal SynRM position control by totally invariant variable structure controller

In this Section, we show the designed procedure for the SynRM position control system which is under the control of the totally invariant variable structure controller.

For position controlled by the LQ method, the system is described by a state-space model. The corresponding SynRM dynamic equations in the state-space model are expressed in eqn. 32

$$\begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_m}{J_m} \end{bmatrix}\begin{bmatrix} \theta_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix}\tau_e - \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix}\tau_l \quad (32)$$

and the electromagnetic equation is given as

$$\tau_e = K_T i_s^2 \sin(2\delta) \tag{33}$$

For a desired rotor position θ_d, one first needs to define the position error and its derivative as

$$\begin{cases} x_1 = \theta_m - \theta_d \\ x_2 = \omega_m \end{cases} \tag{34}$$

Inserting eqns. 33 and 34 into eqn. 32 yields

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} i_s^2 \sin(2\delta) - \begin{bmatrix} 0 \\ \tau_l' \end{bmatrix} \tag{35}$$

where $a = (B_m/J_m)$, $b = (K_T/J_m)$ and $\tau_l' = (\tau_l/J_m)$. Compared with eqn. 12, the corresponding matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are respectively

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{36}$$

Because the LQ method is adopted, under the torque control of MTC for SynRM, one can define the control law as

$$u_L = -\boldsymbol{k}^T \boldsymbol{x} = i_s^2 \sin(2\delta) \tag{37}$$

then the torque equation (eqn. 33) can be rewritten as $\tau_e = -K_T[\boldsymbol{k}^T \boldsymbol{x}]$. First, the nominal condition for SynRM position control will be considered. According to the LQ method, the positive definite matrix $\boldsymbol{Q}$ and positive constant r are first chosen to constitute the performance index (eqn. 2). The choice of elements of matrix $\boldsymbol{Q}$ and positive constant r must take the physical condition for the motor system into consideration. Usually, the smaller r is, the larger the control feedback gain and control law will be. This will accelerate the controlled states toward the set point. For a physical motor control system, the drive system output is always bounded, and in general the large control law u_L would not be realised. Hence, positive constant τ must choose what is physically realisable. For the SynRM drive system, in view of the maximum torque control strategy, the controller output is the command of current magnitude. For the sake of the possibility of physical realisation, the positive constant r will be chosen to prevent the required current command being in the bounded range for a long time.

In the next step, based on the ideal model and the determined matrix $\boldsymbol{Q}$ and positive constant r, the Riccati equation (eqn. 3) is solved to find out the positive definite matrix $\bar{\boldsymbol{P}}$ and feedback gain $\boldsymbol{k}$ calculated on the basis of eqn. 4. Substituting the found feedback gain $\boldsymbol{k}$ into the position control system, the system dynamics can be described as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\boldsymbol{k}^T\boldsymbol{x} = \begin{bmatrix} 0 & 1 \\ -bk_1 & -a - bk_2 \end{bmatrix} \boldsymbol{x} = A_c \boldsymbol{x} \tag{38}$$

where the feedback gain $\boldsymbol{k}$ is defined as $\boldsymbol{k}^T = [k_1, k_2]$. Owing to this feedback control, the dynamical system is a second-order system with a characteristic equation $s^2 + (a + bk_2)s + bk_1 = 0$. The nominal system (eqn. 38) can slide along the predesigned switching surface (eqn. 20) under the control (eqn. 37). That is, the switching surface is always kept at zero throughout the control process and the performance index (eqn. 2) is minimum.

However, uncertain parameters for matrices A and $\boldsymbol{b}$ exist for the physical system, and external load is also existent for most applications of drive systems. Under the influence of uncertainty and disturbance, the perturbed and uncertain system cannot still preserve the system response as the nominal condition, and the system performance under the control of eqn. 37, which is designed for the nominal system, is surely degraded. Moreover, the minimum performance index cannot be achieved and the sliding condition cannot be maintained either. To maintain the sliding condition and preserve the nominal system response and performance subjected to the uncertainty and/or external disturbance, the control law (eqn. 21) must be taken to ensure the existence of the sliding mode. Once the SynRM drive system is controlled by eqn. 21, the perturbed system can be expressed as

$$\begin{aligned} \dot{x} &= (\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^T)\boldsymbol{x} - \boldsymbol{b}q \operatorname{sgn}(\sigma) + \boldsymbol{p} \\ &= A_c\boldsymbol{x} - \boldsymbol{b}q \operatorname{sgn}(\sigma) + \boldsymbol{p} \end{aligned} \tag{39}$$

where A and $\boldsymbol{b}$ are nominal systems, and all the perturbed terms are lumped to the vector $\boldsymbol{p}$ which is $\boldsymbol{p}^T = [0, p]$. As stated, the extra force $-q \operatorname{sgn}(\sigma)$ is used to delete the effects coming from vector $\boldsymbol{p}$, i.e. through this type of control, the system trajectories can be maintained on the sliding mode whether the perturbation is existent or not.

Matrix A_c is the corresponding system matrix under the state feedback control of the nominal system. The perturbed system which is under the control of the control law (eqn. 21) and the switching surface (eqn. 20) will exhibit the same characteristics as the nominal system. The block diagram of the optimal SynRM position control system is shown in Fig. 3.

The designed procedure of totally invariant variable structure control based on optimal SynRM position control is summarised as follows. First, the LQ method is used to design the system characteristics for SynRM position control without considering the uncertainties and disturbances. Thus, the feedback gain $\boldsymbol{k}$, $\boldsymbol{k}^T = [k_1, k_2]$, is chosen

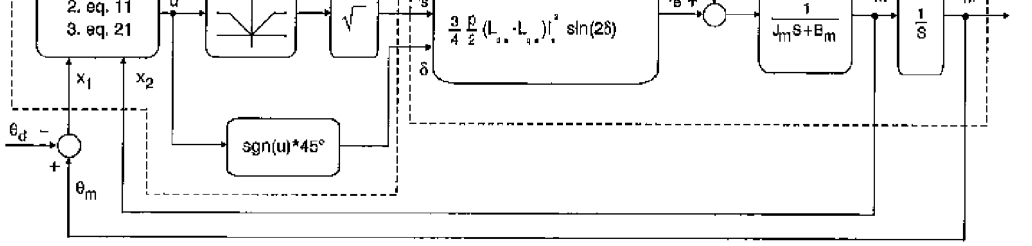

Fig. 3 *SynRM position control system using the maximum torque control strategy*

for the nominal system so that it will be exhibited as a second-order system with characteristic equation

$$s^2 + (a + bk_2)s + bk_1 = 0 \quad (40)$$

where a and b are defined in eqn. 35. In this condition, the system's poles are located at

$$s_{1,2} = \frac{-(a + bk_2) \pm \sqrt{(a + bk_2)^2 - 4bk_1}}{2} \quad (41)$$

and this original system model will yield an optimal performance index.

Secondly, the switching surface $\sigma(x, t)$ of eqn. 20 can be decided as follows. Matrix A_c is the equivalent system matrix under state feedback control for the nominal system with feedback gain $\boldsymbol{k}$. Vector $\boldsymbol{c}^T$ can be simply decided through the choice of $\boldsymbol{c}^T\boldsymbol{b} = 1$. Because the vector $\boldsymbol{b}$ of the SynRM system is $\boldsymbol{b}^T = [0, b]$, vector $\boldsymbol{c}$ can be set as

$$\boldsymbol{c}^T = [0, 1/b] \quad (42)$$

The extra force $-q\,\mathrm{sgn}(\sigma)$ is used to overcome the lumped uncertain parameters and external disturbances. Therefore, the magnitude of q should satisfy

$$q > |p|_{\max} \quad (43)$$

Thus, the sliding condition, $\sigma\dot{\sigma} < 0$ if $\sigma \neq 0$, can be always ensured, and the desired response can also be achieved.

5 Simulation results

Simulations are done by the SIMNON software to verify the proposed control strategy. The parameters of SynRM used in the simulation are given in the Appendix (Section 10). The controlled objective is to drive the motor rotor to rotate 30°. Three different controllers are used to compare the control performances; controllers based on the LQ method and based on the modified LQ method, and the proposed totally invariant VSC-based controller. The dynamical equation of the SynRM drive system (eqn. 35) with parameters shown above is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 12.75 \end{bmatrix}u - \begin{bmatrix} 0 \\ 100\tau_l \end{bmatrix} \quad (44)$$

where $u = i_s^2 \sin(2\delta)$ is adopted.

The nominal system (eqn. 44) under state feedback control with $\tau_l = 0$ is

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -12.75k_1 & -0.2 - 12.75k_2 \end{bmatrix}\boldsymbol{x} = A_c\boldsymbol{x} \quad (45)$$

The corresponding characteristic equation is

$$s^2 + (0.2 + 12.75k_2)s + 12.75k_1 = 0 \quad (46)$$

To decide the feedback gain for the nominal SynRM position control system, matrix $\boldsymbol{Q}$ and positive constant r must first be determined. For the system in eqn. 45, matrix $\boldsymbol{Q}$ and positive constant r are selected to be of the form

$$\boldsymbol{Q} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \; r = 0.1 \quad (47)$$

After substituting eqn. 47 into eqn. 3, the resultant solution $\bar{\boldsymbol{P}}$ is substituted into eqn. 4. Then, the feedback gain $\boldsymbol{k}$ is determined to be the value $[31.62, 31.68]^T$.

As regards the controller based on the modified LQ method, an integral action is going to be added to the original system. To decide the feedback gain, matrix $\boldsymbol{Q}_1$ and positive constant s are chosen to be

$$\boldsymbol{Q}_1 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad s = 0.1 \quad (48)$$

and the resultant feedback gain vector is $\boldsymbol{k}_1^T = [31.62, 33.39, 29.18]$. The system controlled by the modified LQ method based controller has a system order of three, and it is expected that the system will show a slow response differing from that created by the LQ method due to the injected integral action. Of course, the added integral action has the ability to reduce the steady-state error to zero, and this ability is superior to the controller without integral action.

To consider the system design via the proposed totally invariant variable structure control based controller, it preserves the feedback gain obtained at the design stage of the LQ method. In addition, an auxiliary switching surface $\sigma(x, t) = \boldsymbol{c}^T[x - x_0] + A_c\int x d\tau$ and an extra force $-q\,\mathrm{sgn}(\sigma)$ are also added to the control system to build the control scheme of an invariant control system. Thus, the design flexibility on the response requirement is as easy as for the LQ method. Above all, the proposed new method has the ability to reject the influences resulting from the external disturbance and parameter uncertainty, but not for the controller based on the LQ method. In the following, we show the control effects by simulation.

Simulated results for the nominal system are presented in Fig. 4, which shows the position responses for the three different controllers. Owing to the added integral action, the response trajectory shown by the modified LQ method with integral feedback is different from the responses controlled by the other two controllers, and the other two are identical. This means that the trajectory controlled by the totally invariant. VSC is totally matched to the nominal system, and this is what is desired. From Fig. 4, it is evident that if one wants the system controlled by the modified LQ control with integral feedback to have a response similar to that from the LQ method, then some trial-and-error procedures may be needed for the modified LQ method, but it is not necessary for the proposed new method to reach the goals set.

The effects resulting from external disturbances and uncertain parameters are given in Fig. 5. In these simulated results, a 1.0 Nm load is suddenly added to the position

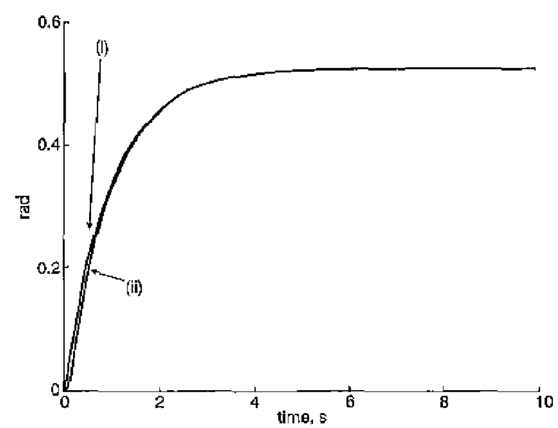

Fig. 4 *Simulated results of SynRM position control system without adding load*

(i) LQ method and totally invariant VSC method, (ii) modified LQ method

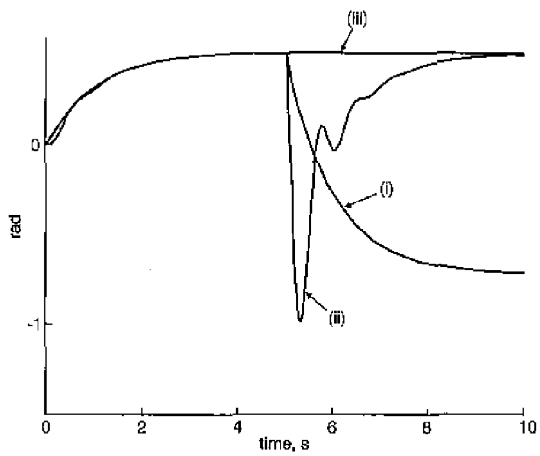

Fig. 5 *Simulated results of SynRM position control system parameter uncertainty $\Delta J_m = 0.04$ Nm/s^2 added load 1.0 Nm at time 5 s*

(i) LQ method, (ii) modified LQ method, (iii) totally invariant VSC method

control system at time 5 s from the beginning, and an uncertain parameter change of J_m from 0.01–0.05 Nm/s^2 is assumed. It is obvious that the position responses show a steady-state error for the controller designed by the LQ method without integral action, but the steady-state error is null for the system based on the modified LQ method owing to the integral action. Beside the fact that the steady-state error is reduced to zero, the system is still affected by the disturbance at the instant the disturbance is added. Looking at the response controlled by the proposed new method, which has an appropriate extra control force q, the system response is independent of the disturbance as Fig. 5 shows. Above all, the responses are the same as the nominal system response before and after the load is added. This proves that the controlled system is invariant to the disturbance.

It is important to consider the hitting condition of the sliding mode control. As the introduction for the totally invariant variable structure control has indicated, system controlled by this new method directly goes into the sliding phase. It means that the system is robust lasting for the control process. To verify this property, a 1.0 Nm load is suddenly added at the instant of the start of control and then removed at time 6 s. Fig. 6 shows the results of this control. The two controllers, based on the LQ method and

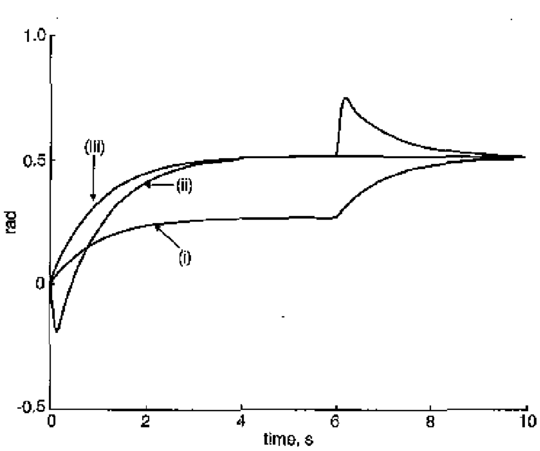

Fig. 6 *Simulated results of SynRM position control system load added at time 0 s, removed at time 6 s*

(i) LQ method, (ii) modified LQ method, (iii) totally invariant VSC method

the modified LQ method, have the worse position responses at these two critical times during which the load is added and removed. However, the system controlled by the proposed new method demonstrates an excellent position response whether the load is added or removed. As the meaning of 'totally invariant' is implied, the system controlled by this new method completely matches the designed nominal system response which is simply determined and designed by the LQ method, and is independent of the external load.

6 Experimental setup and results

6.1 Experimental setup

To practically evaluate the actual performance of the proposed control scheme, a prototype PC-based synchronous reluctance motor position control system was built and tested. The realised system is composed of a Pentium PC, a 12 bit D/A converter, a 12 bit A/D converter, a 1.5 Hp synchronous reluctance motor and a hysteresis current controlled inverter. The position control algorithms are implemented by a Pentium-166 PC. The position signals are sensed by a 2000 pulse/rev encoder and are fed back to the PC through a 16 bit up/down counter. The corresponding mechanical velocity is computed in the PC. To test the feature of the proposed control scheme, a controlled external load disturbance is needed. The SynRM is connected with a brushless DC motor such that a controlled counter torque can be directly added to the SynRM. The main program for managing data input and output is written by the '86 series assembly language and the position control strategies for the three control methods is developed in the mathematical coprocessor language of '387. The experimental data were collected in the PC, processed and printed out through the MATLAB software. The block diagram of this experimental system is shown in Fig. 7.

6.2 Results

To show the validity and effectiveness of the proposed control method, the same position control objects as the simulation is adopted, i.e. a 30° rotor displacement is assumed, and the feedback gain for the modified LQ method is set to be $[31, 33, 26]^T$. However, feedback gain $k = [31, 31]^T$ is set for both the LQ method and the proposed totally invariant variable structure controller. Three conditions all similar to the Section 5 are, respectively, taken into account and executed, and their results are shown and explained in the following.

Fig. 8 shows the position responses for these three controllers in which the external load is absent. In Fig. 8, trace (1) is plotted under the LQ method. Due to the motor uncertain parameters, friction and actuator dead band, etc. the motor system controlled by this simple method will result in a steady-state error. Trace (2) is the response controlled by the modified LQ method. Owing to the integral action, it exhibits a zero steady-state error, and shows a slower rising response compared to trace (3), the response caused by the proposed new controller. To compare the three trajectories with the simulated results in Fig. 4 for the nominal condition, very matched results besides the results caused by the LQ method are shown in Fig. 8, which is affected by the uncertainty and is not considered in the simulation. It is obvious that the desired responses can be easily obtained by the proposed new controller whether the uncertainty exists or not.

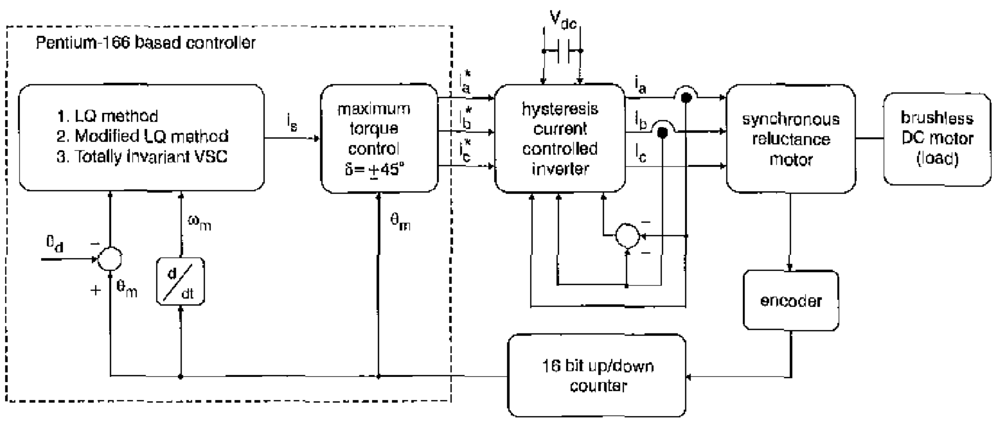

Fig. 7 *PC_166-based SynRM position control system*

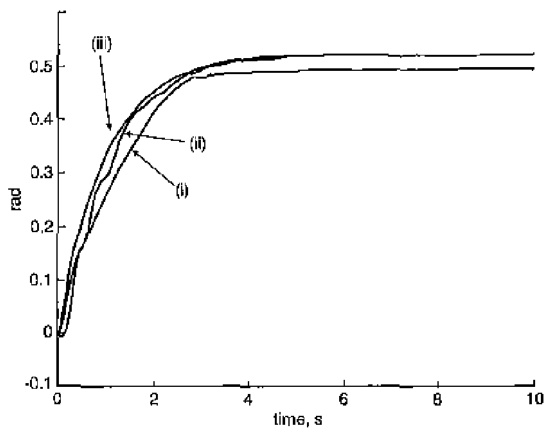

Fig. 8 *Experimental results of SynRM position control system without adding load*

(i) LQ method, (ii) modified LQ method, (iii) totally invariant VSC method

Load effects of these three control methods are shown in Figs. 9 and 10. As in the setting of the simulations, a 1.0 Nm load is suddenly injected to the position system at time 5 s. Fig. 9 shows the responses controlled by the LQ method; the LQ method is based on the nominal system, and once the external disturbance is presented, the steady-state error will outcome as the trajectory regardless of the existence of uncertain parameters. This proves that the poor robust properties of the LQ method will restrict the field of application. In Fig. 10, the position responses controlled by the modified LQ method are shown. As expected, due to the integral action, the steady-state error can be reduced to zero. However, the response trajectory at time 5 s is poor compared to the LQ method. Fig. 11 shows the responses controlled by the proposed new controller. A proper choice of extra force can cancel the effects arising

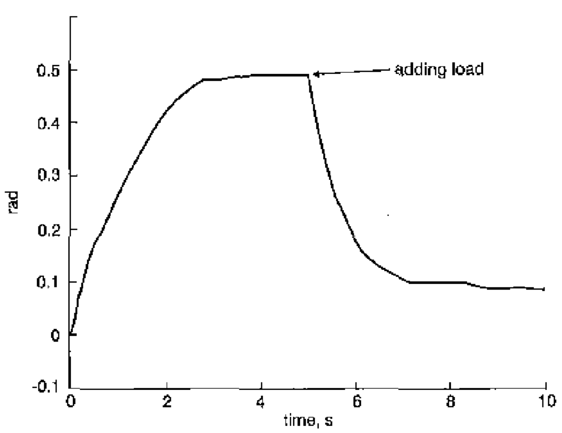

Fig. 9 *Experimental results of SynRM position control system adding load 1.0 Nm at time 5 s*

LQ method

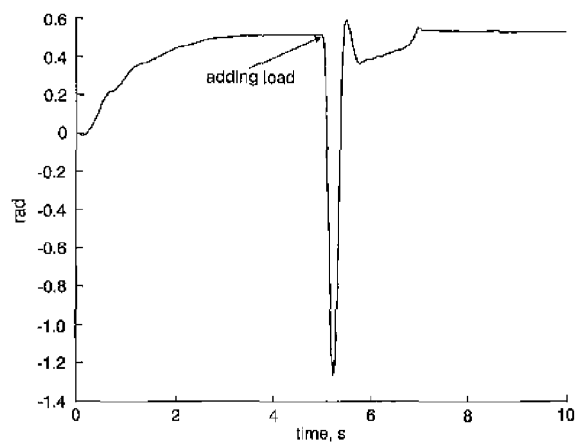

Fig. 10 *Experimental results of SynRM position control system adding load 1.0 Nm at time 5 s*

modified LQ method

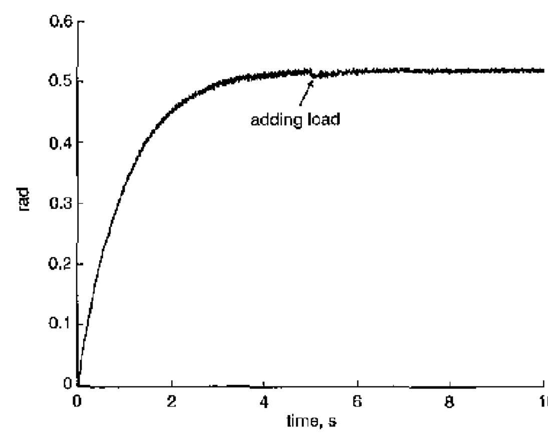

Fig. 11 *Experimental results of SynRM position control system adding load 1.0 Nm at time 5 s*

totally invariant VSC method

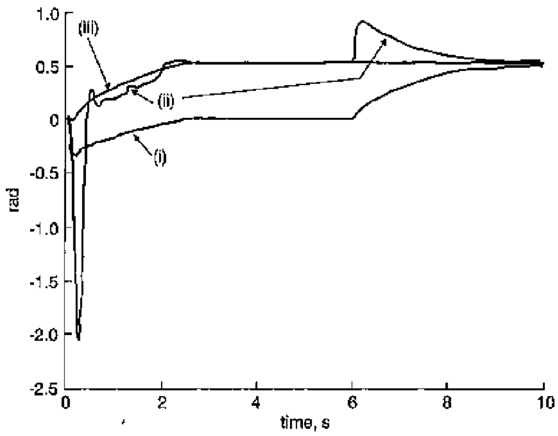

Fig. 12 *Experimental results of SynRM position control system load added at time 0 s, removed at time 6 s*
(i) LQ method, (ii) modified LQ method, (iii) totally invariant VSC method

from the external load, and the responses completely match those desired.

As regards the problem of the hitting phase for variable structure control, a load 1.0 Nm will be added to the experimental system by the brushless DC motor at the starting instant of the experimental process and will be removed at time 6 s to check the invariant property for the proposed controller. This condition is also applied to the other two control methods to evaluate the performance. There are three traces in Fig. 12 demonstrating these results. In Fig. 12, the traces (1) and (2), which are the results from the LQ method and the modified LQ method, respectively, are much affected by the external load at the instants when the load is added and removed. Above all, for the sake of removing the steady-state error, the modified LQ method has a more serious undershoot and overshoot as compared with the LQ method. These effects from the load do not occur for the condition controlled by totally invariant VSC. In particular, the system is in the sliding phase throughout the control process, and completely overcomes the effects resulting from the external disturbance and preserves the desired responses as does trace (3).

7 Conclusions

In this paper, an optimal control scheme is developed for synchronous reluctance motor position control based on the totally invariant VSC. The proposed optimal controller as well as the control scheme, has been demonstrated to be useful in the application of motor position control. It shows that the designed control system fully satisfies the designed requirements whether the uncertainties and disturbances are present or not. The effectiveness of the proposed optimal control scheme and its application to SynRM position control have been demonstrated and verified by both simulation and experiment.

8 Acknowledgment

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9 References

1 SHYU, K.K., and SHIEH, H.J.: 'A new switching surface sliding-mode speed control for induction motor drive systems', *IEEE Trans.*, 1996, **PE-11**, (4), pp. 660–667
2 SHYU, K.K., and SHIEH, H.J.: 'Variable structure current control for induction motor drives by space voltage vector PWM', *IEEE Trans.*, 1995, **IE-42**, (6), pp. 572–578
3 ANDERSON, B.D.O., and MOORE, J.B.: 'Linear optimal control' (Prentice-Hall, 1971)
4 XU, L., and YAO, J.: 'A compensated vector control scheme of a synchronous reluctance motor including saturation and iron losses', *IEEE Trans.*, 1992, **IA-28**, (6), pp. 1330–1338
5 XU, L., XU, X., LIPO, T.A., and NOVOTNY, D.W.: 'Vector control of a synchronous reluctance motor including saturation and iron loss', *IEEE Trans.*, 1991, **IA-27**, (5), pp. 977–985
6 LAGERQUIST, R., BOLDEA, I., and MILLER, T.J.E.: 'Sensorless control of the synchronous reluctance motor', *IEEE Trans.*, 1994, **IA-30**, (3), pp. 673–681
7 BETZ, R.E., LAGERQUIST, R., JOVANOVIC, M., MILLER, T.J.E., and MIDDLETON, R.H.: 'Control of synchronous reluctance machines', *IEEE Trans.*, 1993, **IA-29**, (6), pp. 1110–1122
8 MATSUO, T., ANTABLY, A.E., and LIPO, T.A.: 'A new control strategy for optimum-efficiency operation of a synchronous reluctance motor', *IEEE Trans.*, 1997, **IA-33**, (5), pp. 1146–1153
9 LIU, T.H., and LIN, M.T.: 'A fuzzy sliding-mode controller design for a synchronous reluctance motor drive', *IEEE Trans.*, 1996, **AES-32**, (3), pp. 1065–1075
10 SHYU, K.K., and HUNG, J.C.: 'Totally invariant variable structure control systems'. Proceedings of the IECON '97, New Orleans, USA, pp. 1119–1123
11 YOO, D.S., and CHUNG, M.J.: 'A variable structure control with simple adaptation laws for upper bounds on the norm of the uncertainties', *IEEE Trans.*, 1992, **AC-37**, (6), pp. 860–865

10 Appendix

Motor Data:

rated power	= 1120 W
rated voltage	= 230 V
rated current	= 6.6 A
direct inductance L_{ds}	= 135 mH
quadrature inductance L_{qs}	= 50 mH
stator resistance R_s	= 0.91 Ω
inertia J_m	= 0.01 Nm/sec^2
viscous coefficient B_m	= 0.002 Nm/sec
rated speed	= 1800 rpm